JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 33, No. 2, March–April 1996

Engineering Notes

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Model for Predicting Hypersonic Laminar Near-Wake Flowfields

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Introduction

A RE-ENTRY vehicle is sheathed in a plasma when ionization is present within the shock layer or when ablation of the thermal protection system injects an element having a low ionization potential into the boundary layer along the body surface. In either case, ions and free electrons in the boundary layer are eventually swept behind the body to form a plasma trail that may extend as much as hundreds of body diameters downstream. The distribution of observables in the far wake is strongly dependent on the physical processes occurring near the base of the body. For example, it has been shown that the prediction of the free-electron concentration in the far wake can be changed by orders of magnitude by a modest variation in the recovery enthalpy near the wake stagnation point.[1] Thus an accurate model of the near-wake region is important to a prediction of far-wake properties.

The hypersonic near-wake flowfield has received a considerable amount of theoretical and experimental attention in the literature. An excellent review of the early work done in this area is given in Ref. 2. During the last decade, efforts to understand the near-wake flowfield have centered around a computational fluid dynamics (CFD) approach using Navier–Stokes solvers.[3–5] Although significant fundamental knowledge can be achieved via these CFD-type analyses, there still exists a need for a rapid engineering design tool to predict the effect of ablation products on hypersonic near-wake flowfields. The purpose of this Note is to present a simple engineering model that can be used to predict flowfield properties such as enthalpy and atomic composition in the laminar near-wake region of slender ablating bodies.

Model Description

A knowledge of the hypersonic flowfield over the vehicle surface is required to obtain the necessary conditions to compute the flowfield in the near-wake region. The conditions upstream of the separation point are determined from a simple inviscid/boundary-layer model. Once the freestream properties (Mach number M_∞, Reynolds number Re_∞, and temperature T_∞) and the body geometry (nose radius R_N, cone half-angle θ_c, and base diameter D) are defined, the local boundary-layer edge properties are computed in the following manner. The first step involves determining the asymptotic sharp-cone values for surface pressure p_{sc} and oblique shock-wave angle β_{sc}. Bilinear interpolation of data tabulated in Ref. 6 as functions of M_∞ and θ_c is used to compute p_{sc} and β_{sc}.

Received March 20, 1995; revision received Oct. 2, 1995; accepted for publication Oct. 3, 1995.

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Once p_{sc} is known, the local edge pressure p_e can be calculated by using the following expression:

$$p_e = \frac{\gamma^{\gamma/(2-\gamma)}}{2^{[(4-\gamma)/(2-\gamma)]}}\sqrt{\frac{\pi}{2}}\,p_\infty M_\infty^2\sqrt{C_D}\left(\frac{x}{R_N}\right)^{-1} + p_{sc} \tag{1}$$

Note that the first term in Eq. (1) accounts for bluntness effects by using blast-wave theory and that the value of C_D is estimated by using modified Newtonian theory. Since β_{sc} is also known, the edge velocity u_e can be computed from

$$u_e = \frac{u_\infty \cos\beta_{sc}}{\cos(\beta_{sc} - \theta_c)} \tag{2}$$

The value of u_e can then be used to compute the edge enthalpy h_e by

$$h_e = h_\infty + \tfrac{1}{2}\left(u_\infty^2 - u_e^2\right) \tag{3}$$

The edge temperature T_e is determined by solving a nonlinear equation of the form $h = h(T)$. Utilization of the perfect-gas law then allows one to calculate the edge density ρ_e.

Once the inviscid edge conditions are known, the total mass flow within the boundary layer $\dot{m}$ may be computed by using the following relationship that assumes no interaction between the injected mass and the mass in the undisturbed boundary layer[7]:

$$\dot{m} = 2\pi r(\delta - \delta^*)\rho_e u_e + 2\pi\int_0^x r\dot{M}(\xi)\,\mathrm{d}\xi \tag{4}$$

where $\dot{M}(\xi)$ is the local mass flux resulting from ablation. By using a momentum-integral approach and assuming a linear velocity profile, one can show that the laminar boundary-layer thickness δ for a sharp cone is given by[8]

$$\frac{\delta\sqrt{Re_x}}{x} = \sqrt{\frac{2}{3\alpha}\left(\frac{\mu_w}{\mu_e}\right)} \tag{5}$$

In Eq. (5), the factor of 3 is a result of applying the supersonic cone rule, the constant α is equal to θ/δ, and the viscosity ratio μ_w/μ_e reflects the existence of a temperature variation within the boundary layer. In addition, the laminar boundary-layer displacement thickness δ^* is given by

$$\frac{\delta^*}{\delta} = 1 - \frac{1}{2a}\ln b - \frac{1}{2a}\frac{c}{d}\ln\left[\frac{(d+2a-c)(d+c)}{(d-2a+c)(d-c)}\right] \tag{6}$$

where $a = \frac{1}{2}(\gamma - 1)M_e^2$, $b = T_w/T_e$, $c = 1 + a - b$, and $d = \sqrt{(4ab + c^2)}$.

The structure of the hypersonic near-wake region is shown in Fig. 1. The near-wake region is modeled by employing correlations based on shadowgraph data[9] to define the wake-neck location x_n, the wake-shock origin x_s, the wake-shock angle θ_s, and the wake-neck diameter δ_n. The parameters x_n/D, x_s/D, and θ_s were found to be independent of geometry and Reynolds number over the range $6 \le M_\infty \le 20$ and were approximated by the following least-squares fits:

$$x_n/D = 0.0177M_\infty - 0.248 \tag{7}$$

$$x_s/D = 0.0569M_\infty + 0.159 \tag{8}$$

$$\theta_s = -0.907M_\infty + 30.605 \tag{9}$$

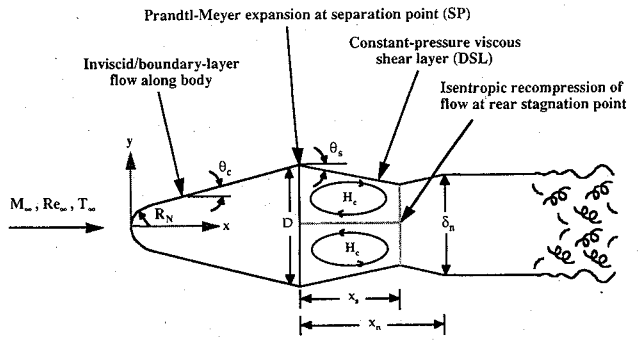

Fig. 1 Re-entry vehicle configuration and base-flow region geometry.

The neck diameter δ_n/D was found to be a function of both M_∞ and Re_∞, and thus a two-dimensional tabular representation of the data is used in this Note. The table lookup that is performed is bilinear with the parameters M_∞ and $Re_{\infty,D}$.

Once the wake geometry is defined, the expansion flow over the separation point can be predicted. The assumption that $(1/u)(\partial u/\partial s) \gg (\sin\theta)/r$ at the separation point allows the pressure change over an axially symmetric corner to be given by two-dimensional Prandtl–Meyer theory.[10] The Mach number downstream of the separation point is determined by solving the nonlinear equation

$$\nu_2(M_2) = \nu_1(M_1) + (\theta_c + \theta_s) \tag{10}$$

where $\nu(M)$ is the Prandtl–Meyer angle corresponding to the Mach number M and the subscripts 1 and 2 refer to points upstream and downstream of the separation point. The pressure and temperature downstream of the separation point are determined from adiabatic, isentropic relations with no change in total properties.

The free shear-layer and core-region flows are predicted by using a modification of the method described in Ref. 11. Briefly, the laminar boundary-layer equations are solved in a coordinate system centered at the separation point with one axis along the dividing streamline and the other axis normal to this. If both the Lewis and Prandtl numbers are taken as unity, the governing conservation equations for momentum, energy, and species can be shown to reduce to the following form:

$$u^* \frac{\partial X}{\partial S^*} = F^{*2} \frac{\partial^2 X}{\partial u^{*2}} \tag{11}$$

where $u^* = u/u_e$, $F = \partial u^*/\partial Y$, $Y = u_e r \int \rho\,dy$, $F^* = \surd(S_w)F$, $S = \int C\rho_e\mu_e u_e r^2\,dx$, and $S^* = S/S_w$. In the above equation, the dependent variable X is either the shear function F^*, the total enthalpy H, or the atomic species concentration c_i. Note that S is the transformed distance measured from the corner separation point and S_w is the transformed length of the wetted vehicle surface. Also, the value of the ratio C/C_w, which appears in the calculation of S^*, is taken to be 0.6. The unknown core-region values of the total enthalpy H_c and the species concentration $c_{i,c}$ are determined by defining new functions E and W, where

$$H - H_e = (H_w - H_e)W + (H_c - H_e)E \tag{12}$$

and

$$c_i - c_{i,e} = (c_{i,w} - c_{i,e})W + (c_{i,c} - c_{i,e})E \tag{13}$$

If the initial conditions are given by the second Crocco energy integral, then E reduces to the quantity $1 - u^* - W$, and the initial and boundary conditions that must be satisfied are

$$E(S^*, 0) = 1, \qquad E(S^*, 1) = 0, \qquad E(0, u^*) = 0$$
$$W(S^*, 0) = 0, \qquad W(S^*, 1) = 0, \qquad W(0, u^*) = 1 - u^* \tag{14}$$

Reference 11 presents solutions to Eqs. (11–13) subject to the above initial and boundary conditions (14) using an implicit finite difference method. In this Note, a tabular representation of these solutions is used to predict the shear-layer flowfield. A logarithmic interpolation on the parameter S^* is performed in the table-lookup procedure to predict the values of F^* and W as functions of u^*.

An overall energy balance is utilized to find the unknown value of the core-region total enthalpy. If there is no mass addition through the base wall into the core region, then one can show that an energy balance over the base-flow region reduces to

$$H_c = H_e - \frac{(H_e - H_w)(K^* - J^*)}{L^*} \tag{15}$$

where

$$\frac{K^* - J^*}{L^*} = \frac{K - J}{L} + \frac{Q}{2\pi L\sqrt{S_w}(H_e - H_w)} \tag{16}$$

and

$$J = \left[\int_{u^*_{\mathrm{DSL}}}^{1} \frac{u^* W}{F^*}\,du^*\right]_n$$

$$K = \left[\int_0^1 \frac{u^*(1-u^*)}{F^*}\,du^*\right]_{\mathrm{sp}} \tag{17}$$

$$L = \left[\int_{u^*_{\mathrm{DSL}}}^{1} \frac{u^* E}{F^*}\,du^*\right]_n$$

The subscripts n, sp, and DSL refer to the values at the neck, the corner separation point, and the dividing streamline, respectively. The core-region total enthalpy H_c and base heat-transfer rate Q are determined by assuming that $J^* = [\phi J]_n$, $K^* = [\phi K]_{\mathrm{sp}}$, and $L^* = [\phi L]_n$. In this Note, the following form of ϕ is chosen:

$$\phi(S^*) = \sqrt{\frac{1 + \ell^*}{1 + S^*}} \tag{18}$$

where ℓ^* is the streamwise distance along the dividing streamline from the separation point measured in units of the streamwise length of the body surface. The nondimensional scaling function $\phi(S^*)$ used here will be shown to yield a good correlation between predicted and measured results over a large range of conditions. A similar analysis involving an overall species balance can be performed to determine the unknown core-region species concentration $c_{i,c}$.

Finally, the flow in the neck region of the wake is predicted by assuming that the mass flow in the wake neck is equal to the total mass flow in the boundary layer on the body. Therefore the law of conservation of mass requires that

$$\dot{m} = (\pi/4)\rho_n u_n \delta_n^2 \tag{19}$$

where $\dot{m}$ is given by Eq. (4). An overall energy balance over the base region yields

$$H_{\mathrm{sp}} = (Q/\dot{m}) + h_n + \tfrac{1}{2}u_n^2 \tag{20}$$

Using the value of u_n from Eq. (19) and the perfect-gas law for ρ_n yields the following implicit equation for the neck temperature T_n:

$$H_{\mathrm{sp}} = \frac{Q}{\dot{m}} + h_n(T_n) + \frac{8}{\pi^2}\left(\frac{\dot{m}R_{\mathrm{gas}}T_n}{p_n\delta_n^2}\right)^2 \tag{21}$$

The value of p_n to be used in Eq. (21) is given by Chapman's point-recompression condition[2]:

$$p_n = p_B \left\{ 1 + \frac{1}{2}(\gamma - 1) \left[\frac{u_{\mathrm{DSL}}^{*2} M_e^2}{1 + \frac{1}{2}(\gamma - 1) M_e^2 \left(1 - u_{\mathrm{DSL}}^{*2}\right)} \right] \right\}^{[\gamma/(\gamma - 1)]} \tag{22}$$

Here, the following correlation based on flight-test data is used to predict the base pressure p_B[12]:

$$p_B = 0.51 p_{\mathrm{sp}} \left[\frac{M_{\mathrm{sp}}\left[1 + \frac{1}{2}M_{\mathrm{sp}}^2\right]^{\frac{3}{2}} [1 + 4(H_w/H_e)]^{\frac{3}{2}}}{Re_{\mathrm{sp},R_B}} \right]^{0.4} \tag{23}$$

Comparison of Model Against Data

A limited amount of experimental data is available to validate two key parameters associated with the present near-wake model. In this Note, comparisons between predicted and measured values of the core-region stagnation temperature and the base heat-transfer rate for slender nonablating cones are shown. Because the thermal resistance of the base-wall boundary layer is not negligible, the core-region enthalpy is always higher than the wall enthalpy for a given heat flux. This fact is illustrated in Fig. 2, which displays the results from the present model and those measured by Zakkay and Cresci[13] for a 5-deg half-angle sharp cone model supported by nonobtrusive wires at $M_\infty = 11.8$, $Re_\infty = 6.1 \times 10^5$/ft, and $T_\infty = 59$°R. Also included in this figure for comparison purposes are the results from a nonsimilar separated shear-layer analysis[14] and a series-expansion method.[15] It is seen that the core-region stagnation temperature is strongly dependent on the wall temperature and that the results from the present model provide the best agreement with the experimental data.

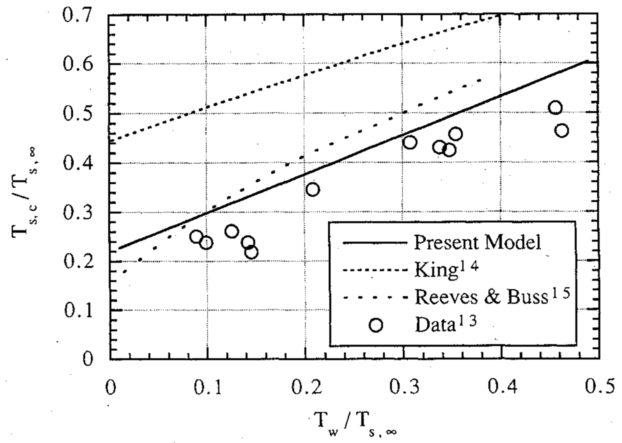

Fig. 2 Core-region stagnation temperature ratio as a function of wall temperature ratio.

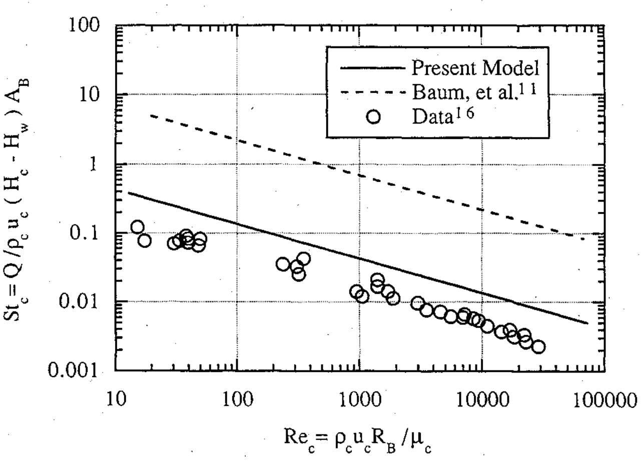

Fig. 3 Core-region Stanton number as a function of core-region Reynolds number.

The most appropriate method of presenting base heat-transfer rate results is to display the core-region Stanton number St_c as a function of core-region Reynolds number Re_c. Figure 3 gives a comparison between the results predicted by the present model and a set of interference-free experimental data[16] that indicate a $Re_c^{-0.5}$ dependence over the range $8 \leq M_\infty \leq 16$. Following Ref. 16, the core-region density–velocity product was taken to be equal to $0.03\rho_\infty u_\infty$. Results from the present model, which were computed for a 5-deg half-angle sharp cone at $M_\infty = 11.8$, $T_\infty = 59$°R, and $T_w/T_{s,\infty} = 0.32$, are also compared with those derived from a potential-flow/boundary-layer model.[11] As seen in Fig. 3, both analytical models predict the same $Re_c^{-0.5}$ dependence, but the present model is in much better agreement with the data.

Conclusions

A simple engineering model has been presented for predicting flowfield properties in the laminar near-wake region of slender ablating bodies. This model consists of an inviscid/boundary-layer flow along the body, a Prandtl–Meyer expansion at the corner separation point, a constant-pressure viscous shear layer separating the outer flow from a stagnant recirculating core region, and an isentropic recompression of the flow at the rear stagnation point. The geometric characteristics of the base-flow region are predicted by using correlations derived from experimental data, and the core-region flowfield properties are determined through overall balance relations. The problem is closed by the introduction of a nondimensional scaling function. For slender nonablating bodies, it is shown that the present model predicts conservative values of the core-region stagnation temperature and the base heat-transfer rate.

Acknowledgment

This work was supported by the Lockheed Missiles and Space Company Independent Development Program.

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